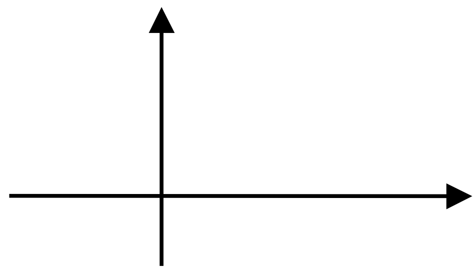
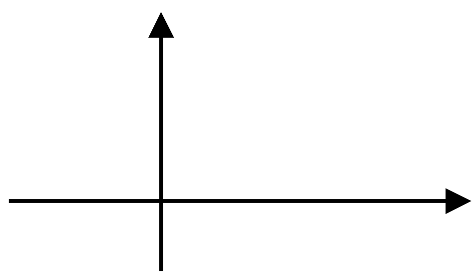
# I. SIGNALS AND SYSTEMS

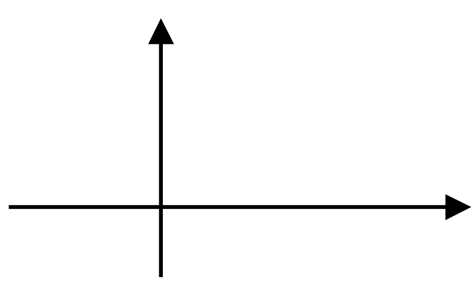
- <u>Signal Type</u>



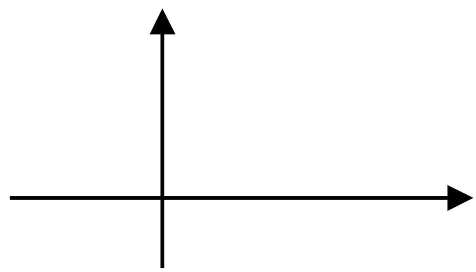
  - Continuous time and continuous amplitude



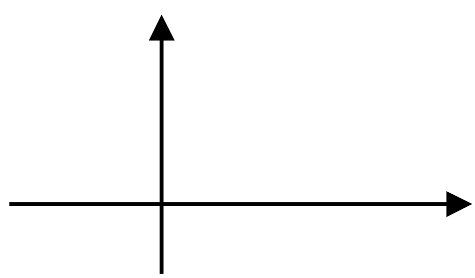
  - Discrete time and continuous amplitude



  - Discrete time and discrete amplitude



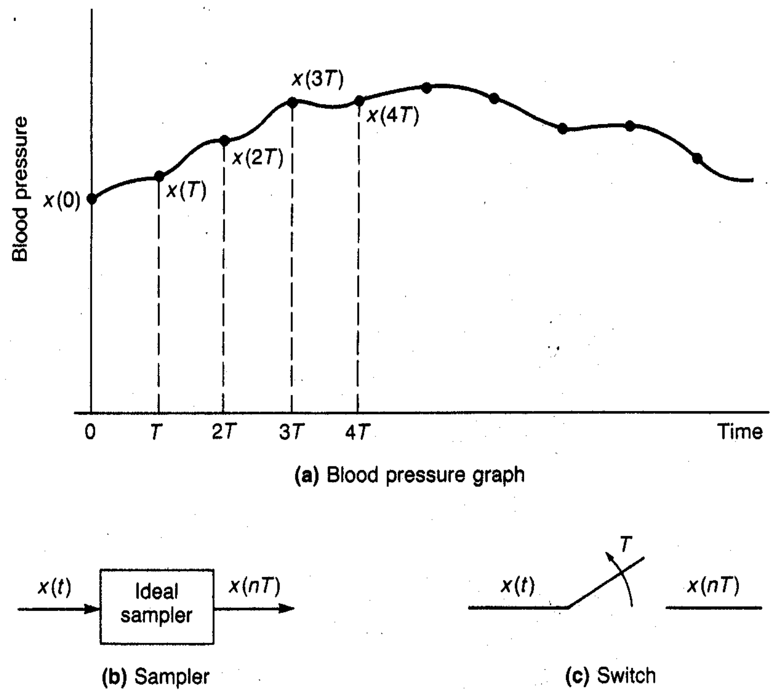
- Continuous time and continuous amplitude with uniform time steps (simplified-data signal)



- Continuous time and discrete amplitude with uniform time-steps

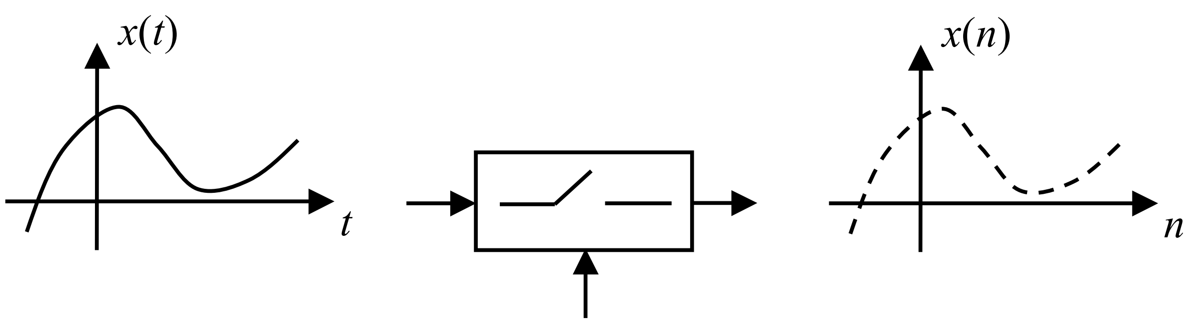
- <u>Sequences</u>

FIGURE 2.3 The sampling process




— Definition

- <u>How to obtain sequences from signals</u>
  - Motivation: Signals are usually processed by a computer. Since the computer understands only numbers and sequences of numbers, the signal has to be converted into a numerical sequence, → i.e., it is sampled

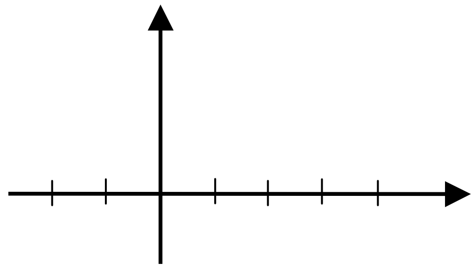



Sampling period: $T$

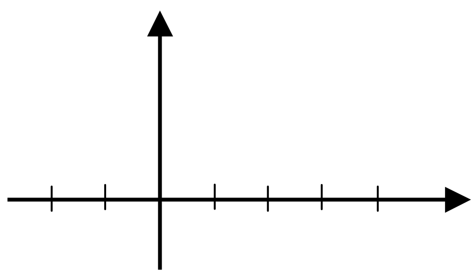
Sampling frequency:

– Basic Sequences

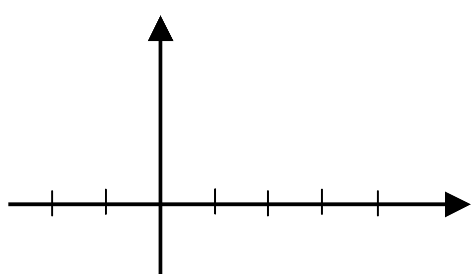
1) Unit impulse sequence



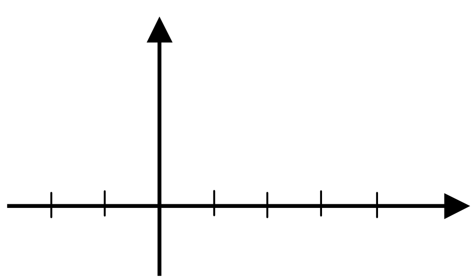
2) Constant sequence



3) Unit step sequence

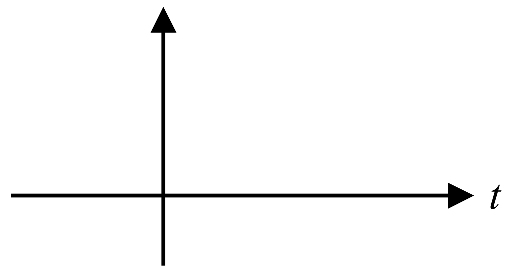


4) Linear sequence

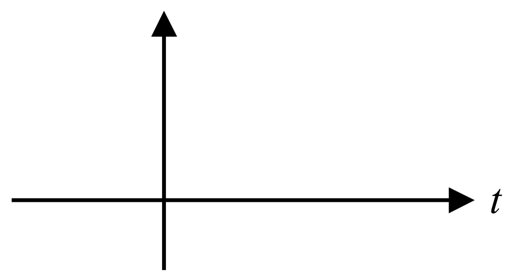


- Basic signals

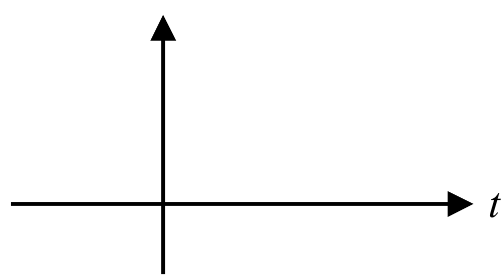
1) Unit impulse signal



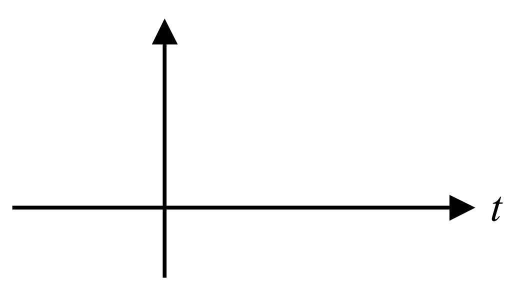

2) Constant signal




3) Unit step signal

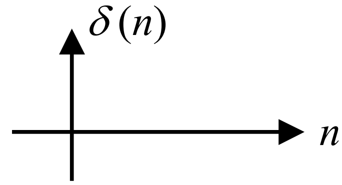



4) Linear signal

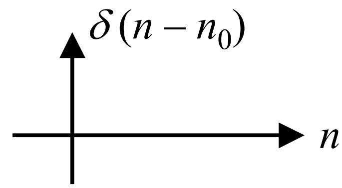


- <u>Signal and sequence shift operations</u>
  - sequence shift

    * Recall $\delta(n)$=

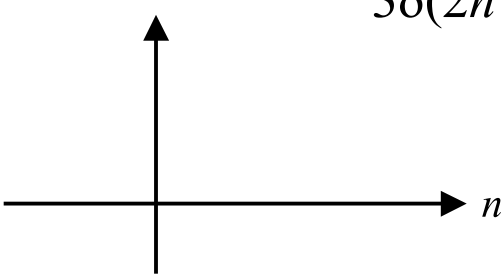


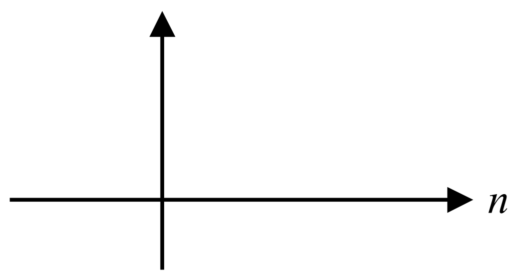

$\delta(n-n_o)$




* Example: plot $3\delta(2n+4)$

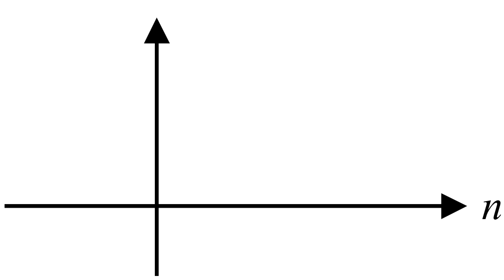
$3\delta(2n-5)$

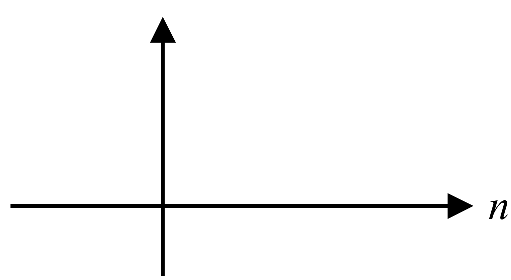


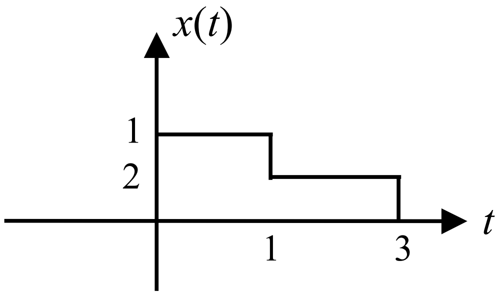

plot $u(n-6)$

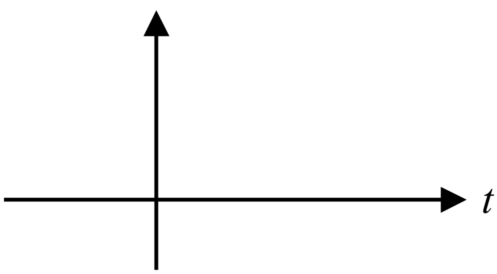
$u(2n+4)$

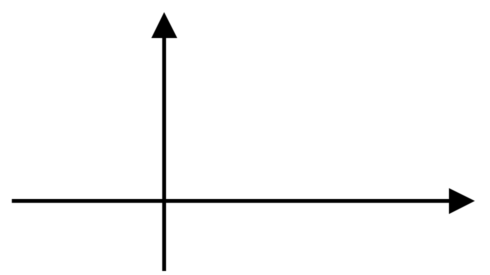


– Signal shift



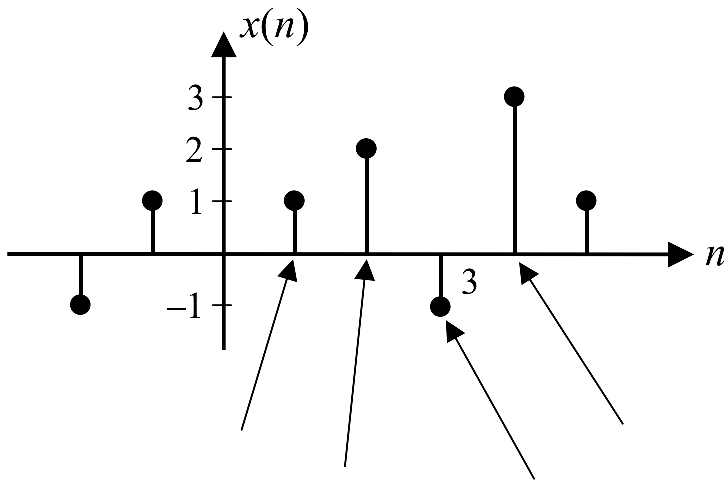




Conclusion: given $x(t)$ representation

$t_o >;\ x(t - t_o)$

$t_o >;\ x(t - t_o)$

– General description of any sequence




$x(n) =$

$\Rightarrow$ any sequence

$x(n)=$

Note: relationship between $u(n)$ and $\delta(n)$

- <u>Periodic Sequences/Signals</u>

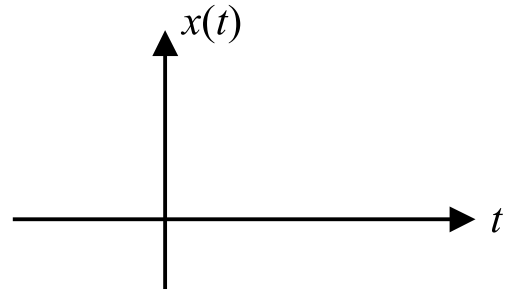
  – Definition: A sequence $x(n)$ is said to be periodic if

  $x(n) =$

  – Definition: A signal x(t) is said to be periodic if

  $x(t) =$

— <u>Sinusoids</u>




* Sinusoidal signal

  $x(t) =$

* Period of $x(t) =$

— <u>Sinusoidal sequence</u>

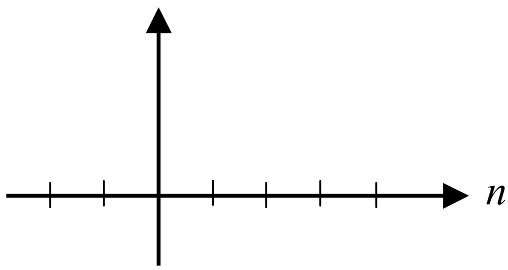
* Assume we sample $x(t)$ with the sampling interval $T_0$

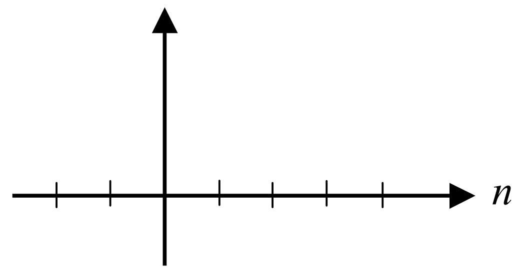
  $x(n) =$

## — <u>Exponential Sequence</u>

1) Real exponential sequence

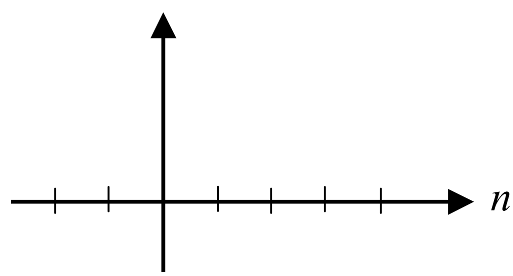
* $x(n) =$




* Commonly used exponential sequence

$x(n) =$




2) Complex exponential sequence (periodic)

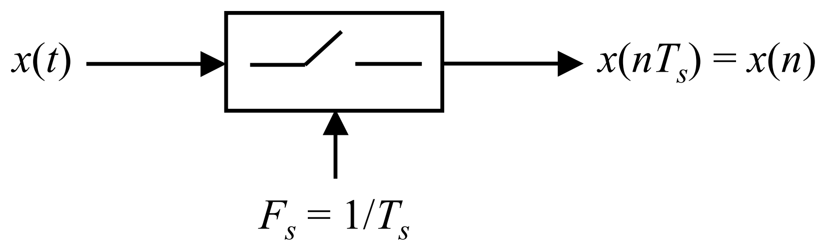
$x(n) =$

* How to plot a complex exponential sequence?

* Is a complex exponential sequence always periodic?

## — Analog and digital frequency

$x(t) =$




* $x(n) = x(nT_s) =$

* Digital frequence $\theta =$

* Question: What is the meaning of the Digital frequency?

* Example: $x(t) = 2\cos(40\pi t + \pi/3)$

$T_s =$

- Plot $x(t)$
- Compute the period of $x(t)$
- Is $x(nT_s)$ periodic?
- Compute the digital frequency

- Relationship between Analog and Digital Frequency Ranges
  - Assume $\theta_0 < \theta < \theta_2$
  - Recall $\theta =$
  - Find the corresponding analog frequency range

- <u>Sampling (Nyquist) Theorem</u>

> — Goal: The sampling theorem indicates how fast one must sample a continuous signal to be able to uniquely represent the continuous signal by its sampled version.

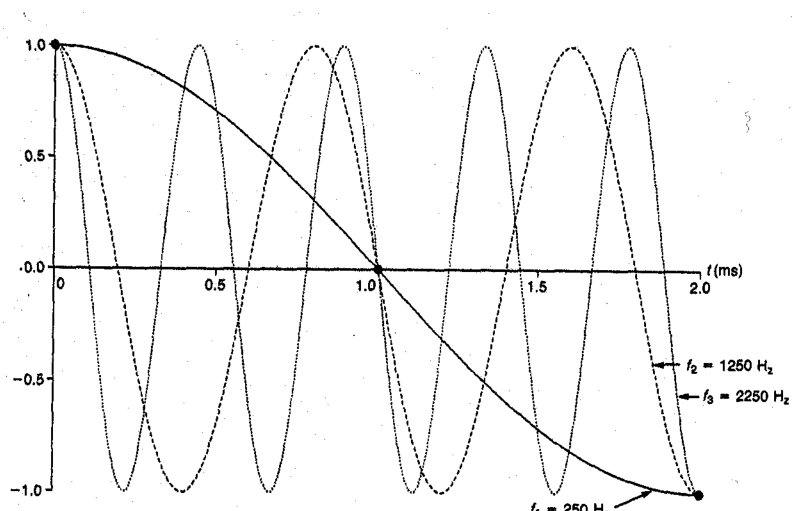



**FIGURE 2.28** Illustration of ambiguity in undersampled signals

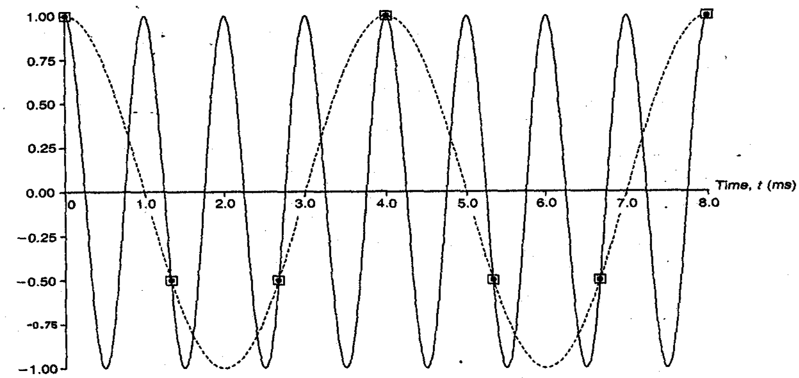



**FIGURE 2.29(a)** Result of under sampling, $f_s$ = 750 Hz or $T$ = 1.33 ms

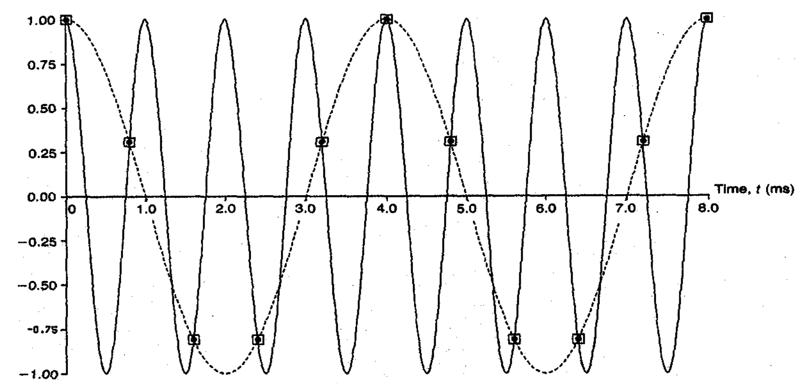



**FIGURE 2.29(b)** Result of under sampling, $f_s$ = 1250 Hz or $T$ = 0.8 ms

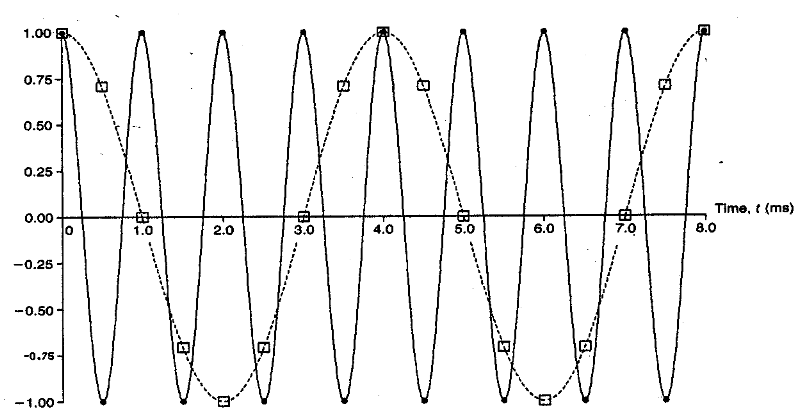



**FIGURE 2.29(c)** Result of sampling at Nyquist rate: sampling period of $T$ = 0.5 ms or $f_s$ = 2000 Hz

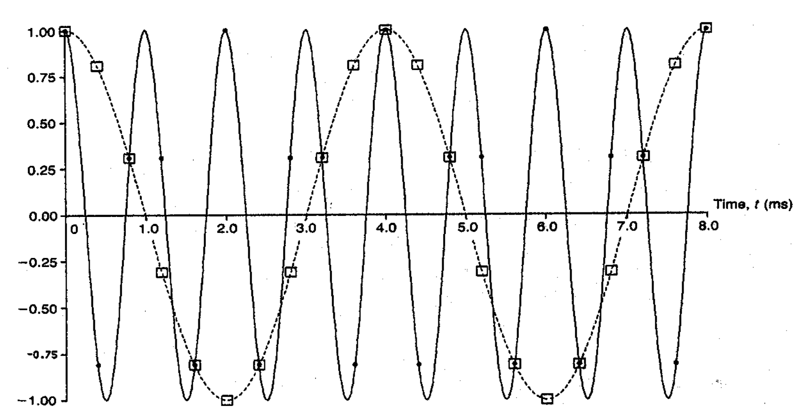
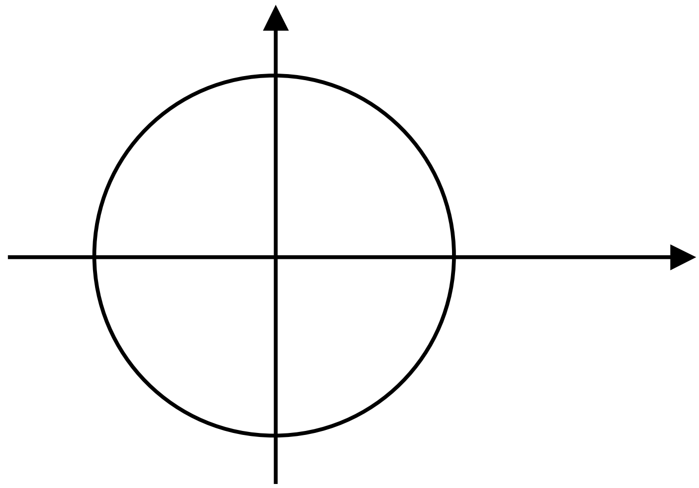



**FIGURE 2.29(d)** Result of sampling above Nyquist rate: sampling period of $T$ = 0.4 ms or $f_s$ = 2500 Hz

* Example: $x(n) = \cos(\theta n)$



* Range of digital frequencies which may be distinguished from each other:

* Application: if $0 < \theta < \pi$

What range does the corresponding analog frequency range have?

## Nyquist Theorem

A sampled signal $x(n)$ can be uniquely represented by equally spaced samples if the sampling frequency $f_s$ is greater than $2f_{\max}$, where $f_{\max}$ is the maximum frequency of the continuous signal $x(t)$ generating $x(n)$.

* Why is the Nyquist theorem important?